GE/AY 133 Problem Set #3

Due April 23

1. Grain Growth and Planetesimal Formation

In class, we discussed the importance of radial drift to the growth of planetesimals. Equation (4.38) of Armitage gives an expression for radial drift rate of a solid particle. In the problems below, you can assume $v_{r,gas}$ to be negligible. To calculate v_r, assume the disk has an aspect ratio of h/r=0.05. The drift timescale is given by $t_{drift} = r / v_r$ (you can think of this as the time scale for the grain to accrete onto the star). Assume that the solar nebula had a surface density profile that was inversely proportional to the orbital radius and its value at r = 1AU was 2000 g/cm^2.

a. a) Given that a typical disk lives for ~3 Myr, what is the maximal pebble size that can safely survive the entire lifetime of the disk? Suppose the pebble begins at 1 AU. A pebble is defined as having a stopping time τ_{fric} ranging from much less than one to about 1. Assuming $\tau_{fric} << 1$, make an approximation in Equation (4.38) to find the solution. Of course, your answer should correspond to a particle size with $\tau_{fric} << 1$ to validate your earlier approximation. How does this compare with typical chondrule sizes?

b. b) For a planetesimal starting at 1 AU, what is the minimum planetesimal (ie, $\tau_{fric} >> 1$) size that would survive for 3 Myr?

c. c) How much solid material do you expect to have resided in the solar nebula from 0.05 AU to 5 AU, in Earth masses. Assume a dust-to-gas ratio of ~ 0.01. Compare this mass to the mass of the present day terrestrial planets and asteroids. What is the main mechanism responsible for removing this mass?

d. d) Suppose that planetesimals of around 100km manage to form from pebbles. How does this compare your answer to part (b)? Knowing that the region around 1 AU today is mostly cleared of planetesimals, what can you infer about planetesimal interactions in the solar nebula? (Hint: How might the planetesimal size be reduced below your answer to (b)?)

So far, you have found an upper size limit for the survival of pebbles and a lower limit to the survival of planetesimals. A classical problem in the field of planet formation is just how nature got over this “meter-sized barrier,” whereby pebble-sized objects will spiral inward toward the Sun before reaching the size of planetesimals. The streaming instability has emerged as a promising solution to this problem (described in section 4.2 of Johansen_2014.pdf on the course website, or if you like linear stability analysis, check out Youdin & Goodman 2005, where the instability is mathematically derived).

e) Come up with an analogy not involving bicyclists for the streaming instability. Identify at least 3 properties of the streaming instability and describe how your analogy fits these properties.

2. Gravitational Focusing

One of the most important concepts in pairwise accretion is the “gravitational focusing” caused by the deflection of a small body by a massive body. This deflection works to increase the cross-section for collision of the two bodies. Your goal is to reproduce a calculation first done by Safronov. Consider a test particle of mass m approaching a body of mass M and radius R. The impact parameter is b and the velocity at an infinite distance is v (i.e. in the absence of gravity, the two bodies would approach within a distance b of each other and with relative velocity v). If gravity was ignored, the cross-section for collision would be R^2. We would like to know by how much this cross-section increases due to the effects of gravity.

a. a) What is the escape velocity from the large body?

b. b) Draw a diagram of a *grazing* collision between the two bodies. (A grazing collision means that if the point mass had just slightly more energy it would not collide with the massive body). Pay particular attention to where the two bodies touch.

c. c) For the case you just drew, use the principle of conservation of angular momentum to write an equation relating b, v, R, and v_{imp}, the velocity of the object at impact. (Hint: write down the total angular momentum in the starting configuration and the total angular momentum at the moment of impact, and set these equal)

d. d) Write an energy conservation equation relating v, v_{imp}, R, and M.

e. e) What is the maximum value for b for which an impact will occur? Write your answer in the form of $b^2 = R^2(1+f)$ where f is called the focusing or Safronov parameter, which should be expressed in terms of the escape velocity.

f. f) How large does the object need to be before gravitational focusing significantly increases the impact (and thus accretion) cross section? The answer to this question is not obvious without some additional information that you don’t necessarily possess, so do your best and make reasonable estimates where necessary. Justify your answer carefully.

g. g) A related important concept is that of *dynamical friction*, in which large bodies are slowed by the presence of a swarm of smaller bodies. What happens to the orbit of the larger body as its velocity is decreased by these interactions, and how might that affect its growth rate? Conservation of energy tells us that the energy lost by the large body must be transferred to the smaller bodies, resulting in increased velocity dispersion and a higher average velocity. What effect would this increase in average velocity have on the ability of the large body to accrete material?

h. h) In question 2(e), you derived the focusing factor for pairwise accretion of a test mass onto a large body, assuming the test particle acts like a planetesimal (i.e. in Stokes drag regime). Now, consider the cross-section for accretion of a pebble onto a planetesimal. Would this cross-section be the same as the one derived in question 2(e)? Why or why not? If it’s different, what is different about the accretion process that you would have to account for to derive the cross-section for accretion of a pebble? (Hint: a pebble has dimensionless stopping time below unity.)